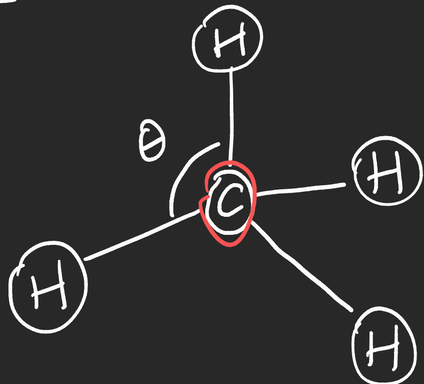
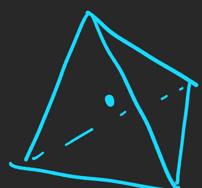
Minor correction to Lec 3's posted slides:

"The component (scalar projection) is, up to sign, the magnitude of the vector projection."

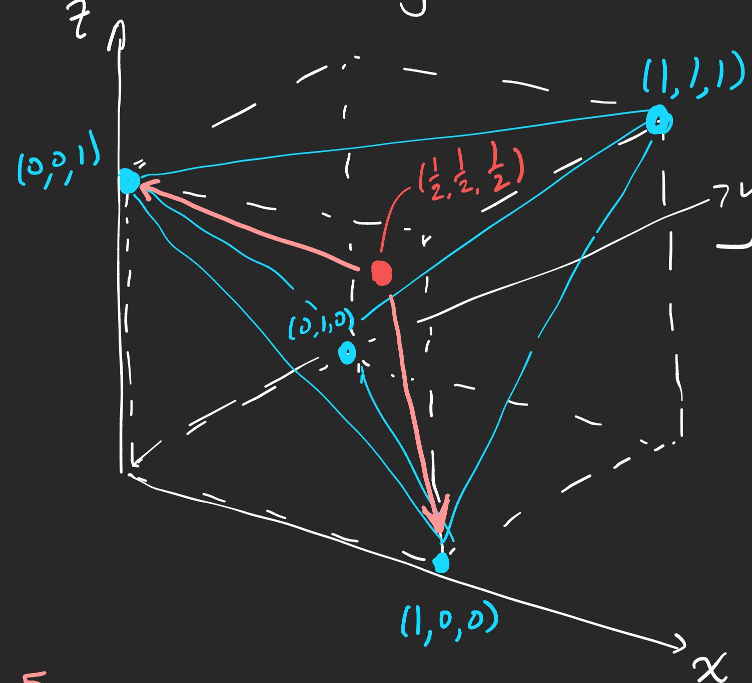
Example §12.3 Ex 57





tetrahedron w/ carbon @ center

Q: What's the bond angle θ?



$\vec{u} =$ = head - tail = $\langle -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2} \rangle$

$\vec{v} =$ = $\langle \frac{1}{2}, -\frac{1}{2}, -\frac{1}{2} \rangle$

$$\cos\theta_{\vec{u},\vec{v}} = \frac{\vec{u}\cdot\vec{v}}{|\vec{u}||\vec{v}|} = \frac{(-\frac{1}{2})(\frac{1}{2})+(-\frac{1}{2})(-\frac{1}{2})+(\frac{1}{2})(-\frac{1}{2})}{\sqrt{\frac{1}{4}+\frac{1}{4}+\frac{1}{4}}\sqrt{\frac{1}{4}+\frac{1}{4}+\frac{1}{4}}}$$

$$= \frac{-1/4}{3/4} = -\frac{1}{3}$$

$$\theta_{\vec{u},\vec{v}} = \cos^{-1}\left(-\frac{1}{3}\right) \approx 109.5^\circ$$

Q: We also have this formula

$$|\vec{u}\times\vec{v}| = |\vec{u}||\vec{v}|\sin\theta_{\vec{u},\vec{v}}$$

i.e. $$\sin\theta_{\vec{u},\vec{v}} = \frac{|\vec{u}||\vec{v}|}{|\vec{u}\times\vec{v}|}.$$

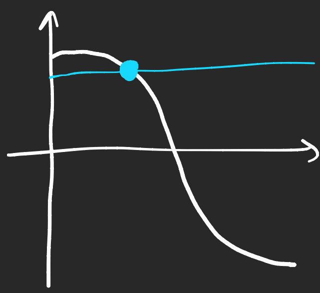
Why don't we use this one to compute the angle?

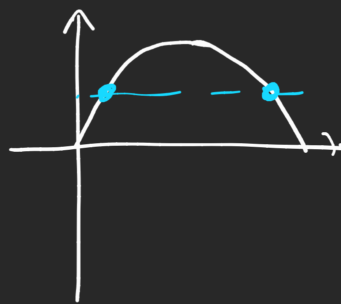
Reason: if you compute

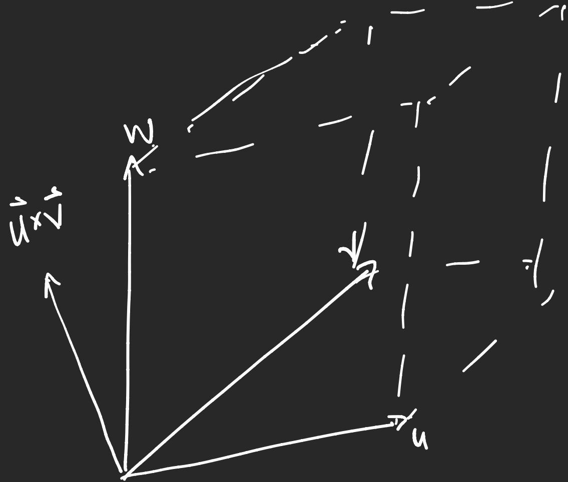
$$\sin\theta = \frac{1}{2} \text{ for example...}$$

That doesn't distinguish between

$$\theta = \frac{\pi}{6} \quad \text{v.s.} \quad \theta = \frac{5\pi}{6}.$$







For all vectors $\vec{w}$, the expression $(\vec{u}\times\vec{v})\cdot\vec{w}$ computes the volume of the above parallelepiped (up to sign).

Ex) · if $\vec{w}=\vec{u}$, conclude $(\vec{u}\times\vec{v})\cdot\vec{u}=0$

similarly

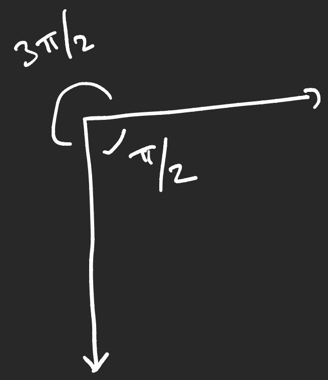
$(\vec{u}\times\vec{v})\cdot\vec{v}=0$.

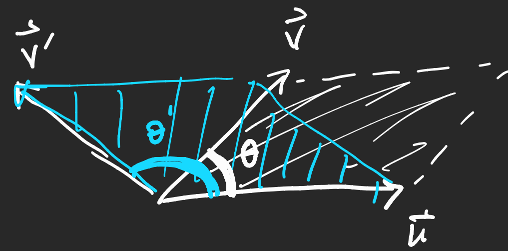
· if $\vec{w}=(\vec{u}\times\vec{v})$ then

$$(\vec{u}\times\vec{v})\cdot(\vec{u}\times\vec{v})=|\vec{u}\times\vec{v}|^2$$

$\|$ by geometric interp.

(Area of parallelogram spanned by $\vec{u},\vec{v}$) $\cdot\,|\vec{u}\times\vec{v}|$





In this situation: $\sin\theta = \sin\theta'$

but $\theta \neq \theta'$.